

A one dimensional hydraulic model for confined navigation with a longitudinal variation of the hull

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Phenomenology & general parameters

$$\text{Re} = \frac{VL}{\nu}$$

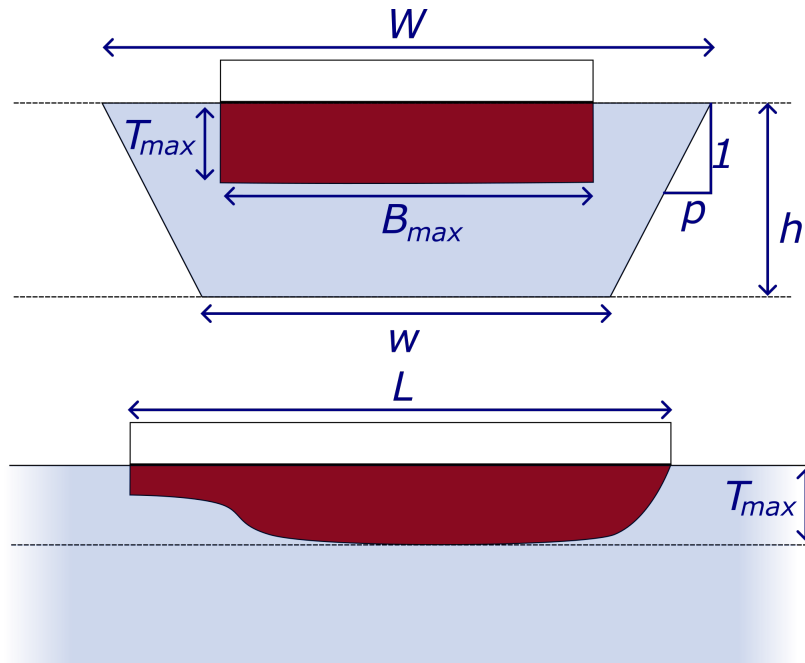
$$\text{Sr} = \frac{B_{\max}}{(W + w)/2}$$

$$\mu = \frac{h}{L}$$

$$\text{Fn}_h = \frac{V}{\sqrt{gh}}$$

$$\text{An} = \frac{T_{\max}}{h}$$

$$\text{Td} = p \frac{h}{W}$$



$$m = \text{Sr} \times \text{An}$$

Turbulent wake

Wave wake



Courtesy: C. Gesing, BAW

Legacy hydraulic models (r-KSC/t-KSC)

Hypotheses:

- No effect of viscosity
- No transverse variations of the flow
- Slow longitudinal variations of the flow

Asymptotics:

- $Re \rightarrow +\infty$
- $Sr \rightarrow 1$ ($p \rightarrow 0$)
- $\mu \rightarrow 0$

Actual ranges:

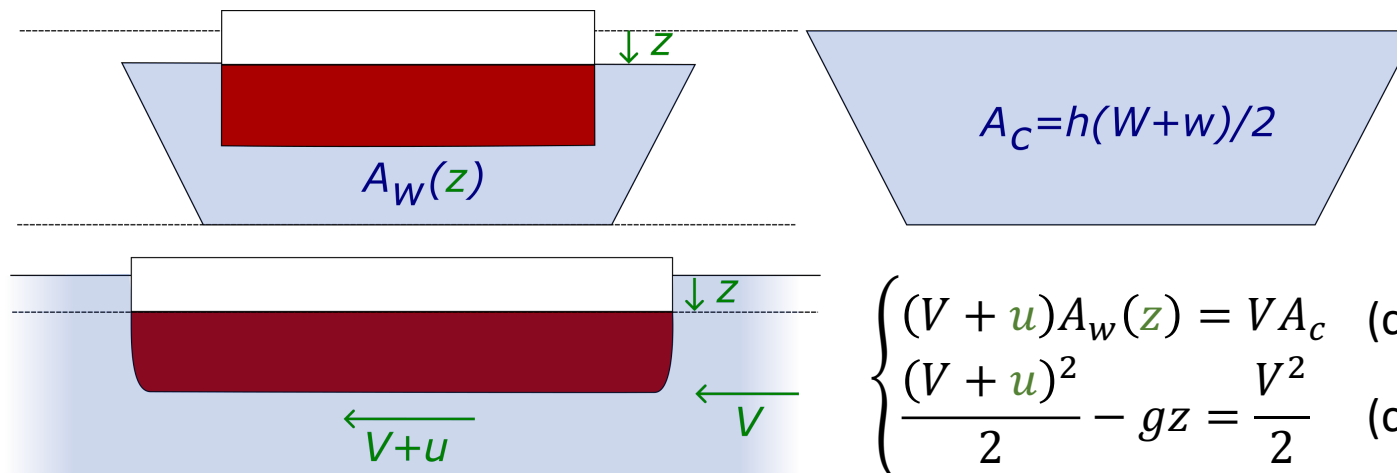
- $Re \sim 10^7$
- $Sr \sim 0.25 - 0.5$
- $\mu \sim 0.04$

Consequences:

- No turbulent wake...
- No flow deviation...
- No wave wake...

...in the models !

Water drawdown/sinkage (z) and return current (u):

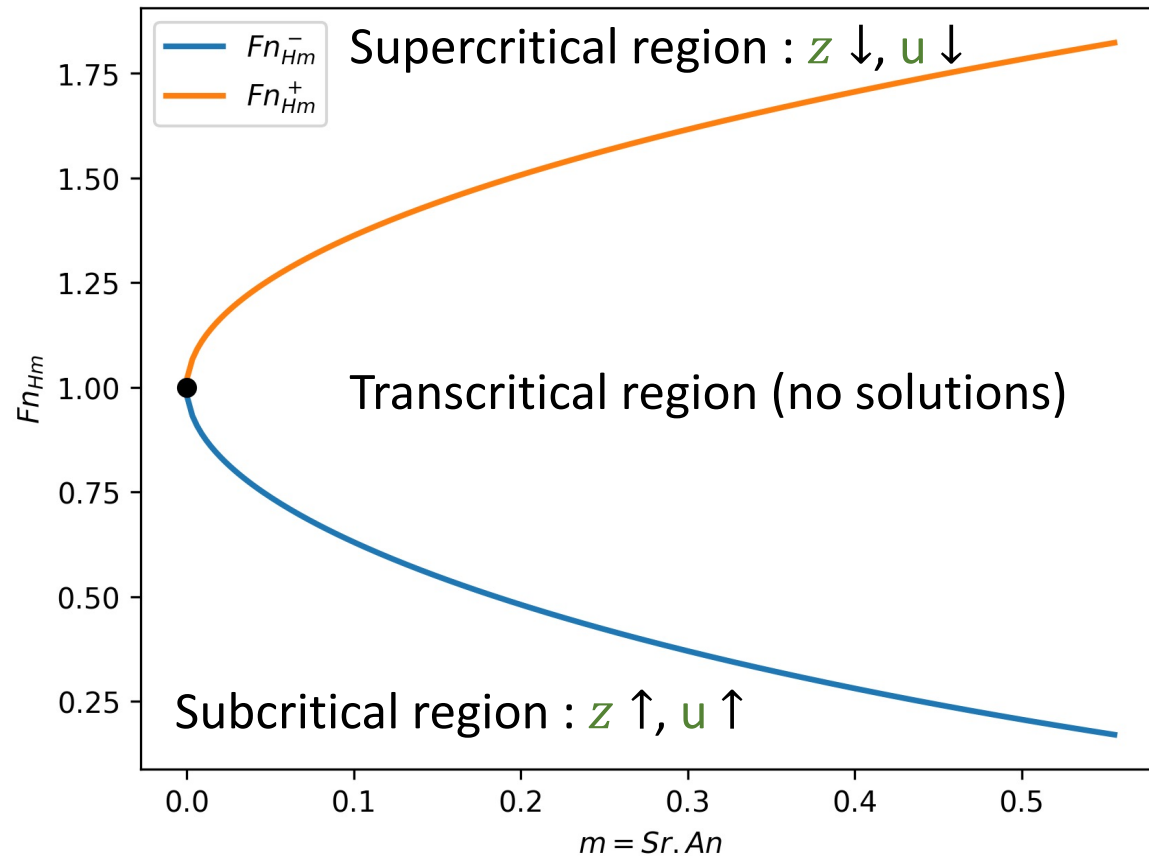


1. Kreitner, J, 1934.
2. Schijf, J.B., 1949.
3. Constantine, T, 1960.

$$\begin{cases} (V + u)A_w(z) = VA_c & \text{(cons. of mass flux)} \\ \frac{(V + u)^2}{2} - gz = \frac{V^2}{2} & \text{(cons. of hydraulic head)} \end{cases}$$

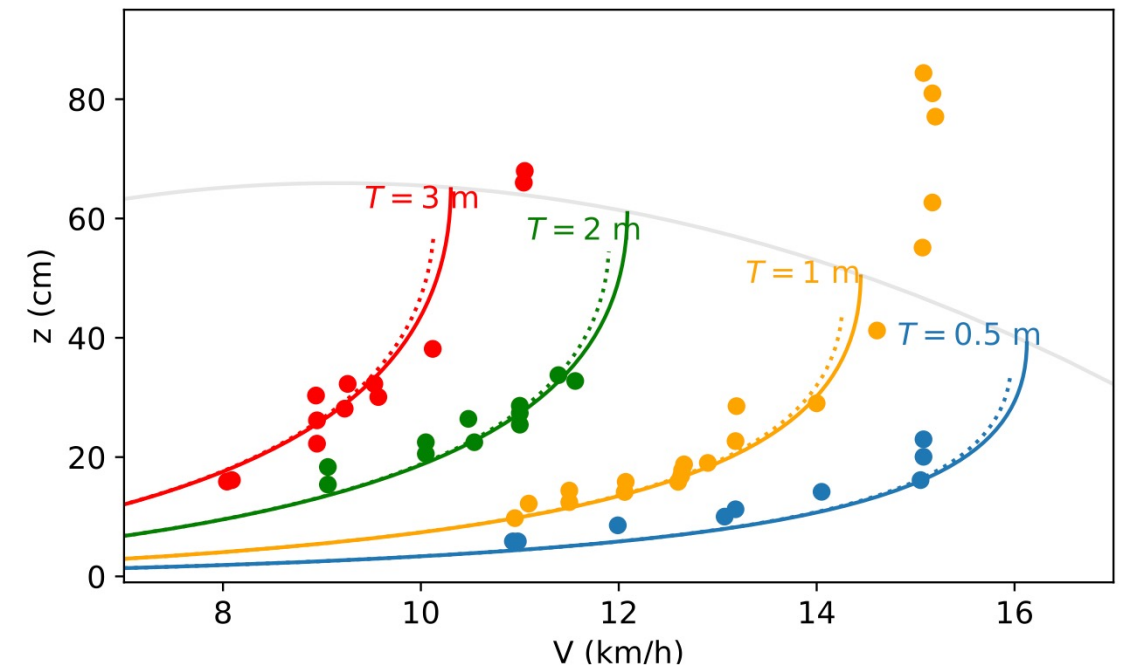
Legacy hydraulic models (r-KSC/t-KSC)

- For a given blockage ratio, 2 critical speeds :



Generalization to a trapezoidal shape

- Comparisons w/ experiments (SOGREAH, 1977)

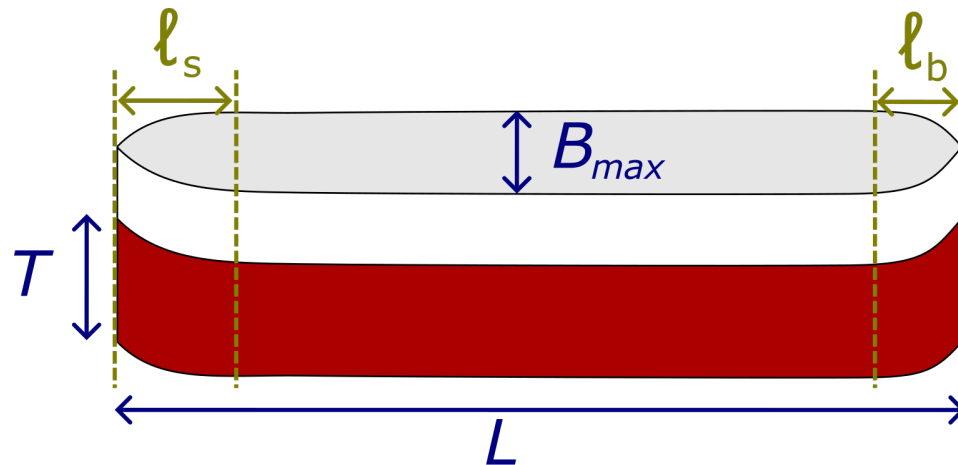


Jules, Valentin, Pierre-Jean Pompée, Julien Dambrine, Morgan Pierre, and Germain Rousseaux, 2024. A revisit of confined medium navigation theories. In: 19e Journées de l'Hydrodynamique, Nantes.

Geometrical parameters of the boat

Block-type boats are in contradiction with *slow flow variations*!

→ **Prismatic** boat with parabolic end profiles:

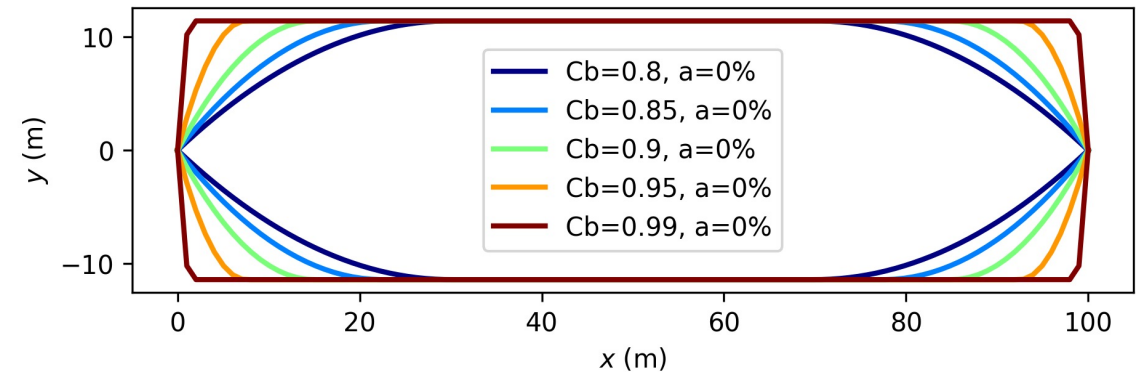
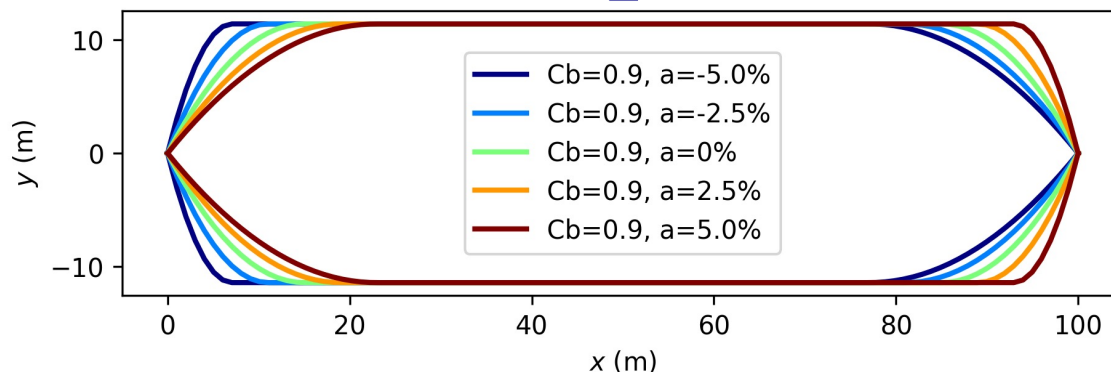


- Block coefficient:

$$C_b = \frac{\int_0^L B(x) dx}{B_{max} L} = 1 - \frac{\ell_s + \ell_b}{3L}$$

- Asymmetry coefficient:

$$a = 1 - \frac{\int_0^L x B(x) dx}{L \int_0^L B(x) dx} = \frac{\frac{\ell_s^2 - \ell_b^2}{6L^2} - \frac{\ell_s - \ell_b}{3L}}{1 - \frac{\ell_s + \ell_b}{3L}}$$



« Boat's ends effect » hydrostatic model

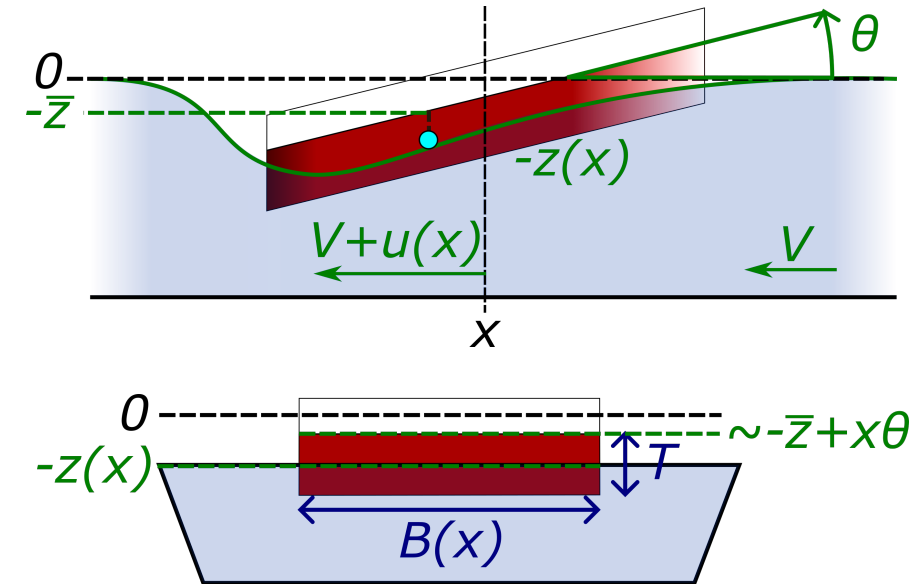
- Variable beam and draught profiles :

→ Sinkage ($\bar{z}$) $\neq$ non-constant water drawdown ($x \mapsto z(x)$)

→ Non-constant return current ($x \mapsto u(x)$)

→ Trim (θ) must be allowed if $a \neq 0$

$$\left\{ \begin{array}{l} (V + u(x))A_w(x, z(x), \bar{z}, \theta) = VA_c \quad \leftarrow \text{(cons. of mass flux)} \\ \frac{(V + u(x))^2}{2} - gz(x) = \frac{V^2}{2} \quad \leftarrow \text{(cons. of hydraulic head)} \\ \int_{x_s}^{x_b} B(x)(\bar{z} - x\theta + T - z(x))dx = 0 \quad \leftarrow \text{(boat's hydrostatic balance)} \\ \int_{x_s}^{x_b} x B(x)(\bar{z} - x\theta + T - z(x))dx = 0 \end{array} \right.$$



$$A_w(x, z, \bar{z}, \theta) = A_c - B(x)(\bar{z} - x\theta + T - z) + z(W - pz)$$

Very close to : E. Lataire, M. Vantorre, G. Delefortrie, *A prediction method for squat in restricted and unrestricted rectangular fairways*, Ocean Engineering (2012).

« Boat's ends effect » hydrostatic model

- Reformulation as a single non-linear integral equation ($\sim$ Kreitner's cubic equation)

$$\left(\frac{2}{\text{Fn}_h^2} z(x) + 1 \right) \left(1 - \text{Sr} B(x) (\bar{z}(B, z) - z(x) + \text{An} T - (x - x^*) \theta(B, z)) - \frac{1}{1 - \text{Td}} z(x) (1 - \text{Td} z(x)) \right)^2 = 1$$

- Fn_h^+ and Fn_h^- now depend on a and C_b (through $x \mapsto B(x)$)
- Mathematical guarantees ($a = 0$, $\text{Td} = 0$)
 - Existence/uniqueness of a subcritical *solution* for a small enough Fn_h
 - *Convergence* towards the r-KSC theory as $\text{C}_b \rightarrow 1$ for a small enough Fn_h
- Numerical approximation of solutions:
 - Fixed-point iterations
 - Approximation of the integral terms with a simple quadrature method

Numerics : the effect of C_b

$a = 0$ (no trim)

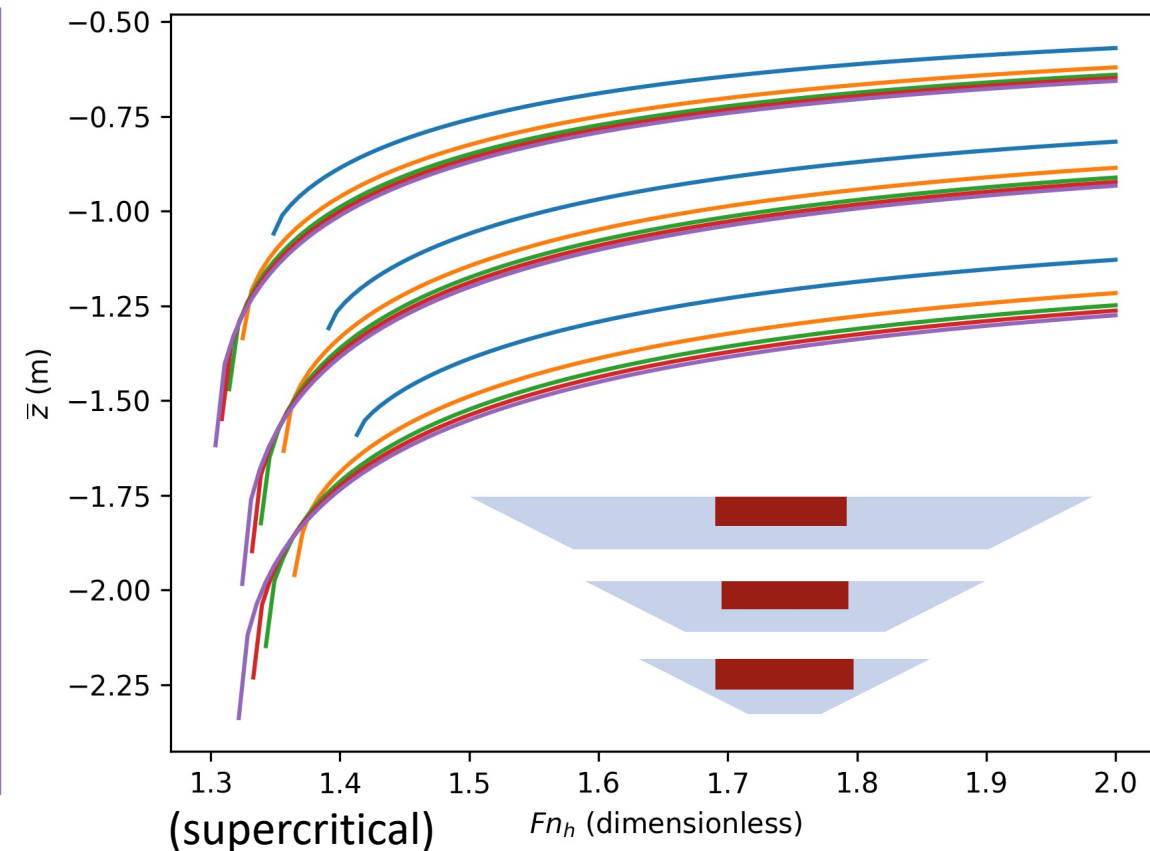
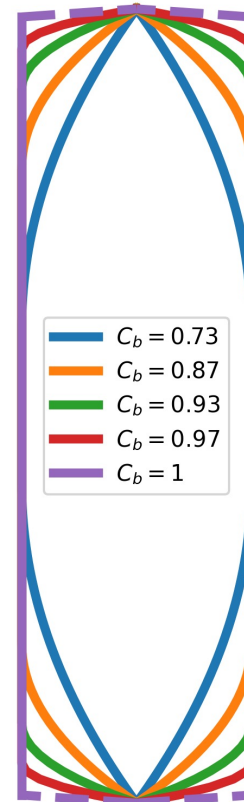
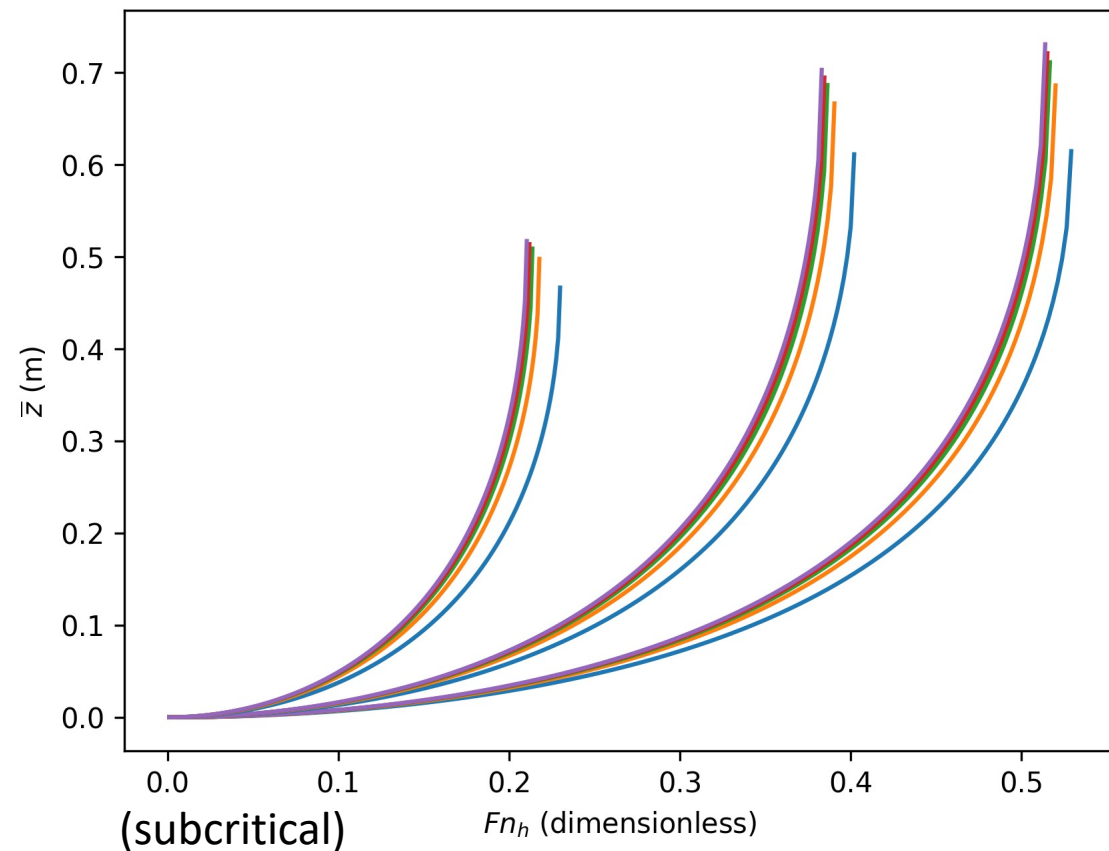
$Sr = 0.76$

$Sr = 0.42$

$Sr = 0.253$

Large Rhine class in a CEMT class-Vb canal

- $B_{max} = 11.4m$
- $T = 2.5m$
- $W = 54m$
- $p = 2$
- $h = 4.5m$



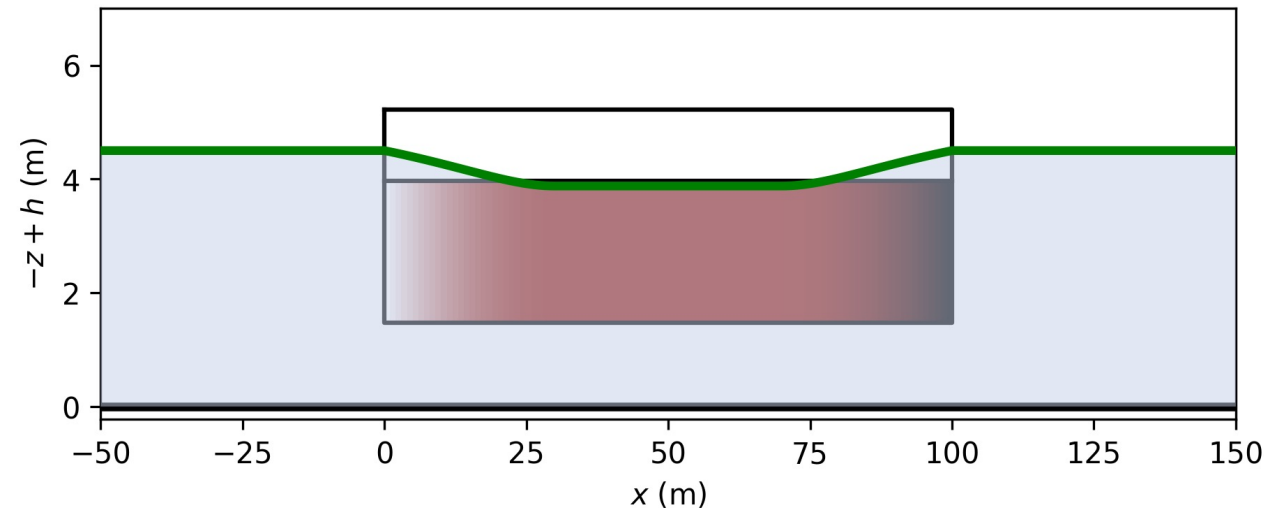
Numerics : the effect of C_b

$a = 0$ (no trim), $C_b = 0.8$

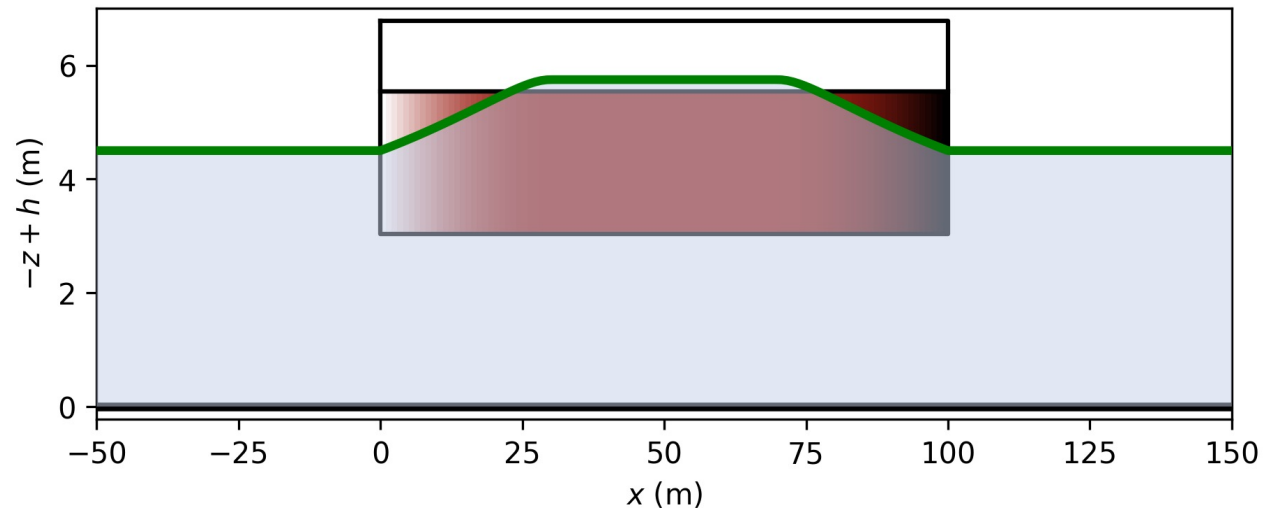
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$$Fn_h = Fn_h^-$$



$$Fn_h = Fn_h^+$$



Numerics : the effect of a

$$C_b = 0.9$$

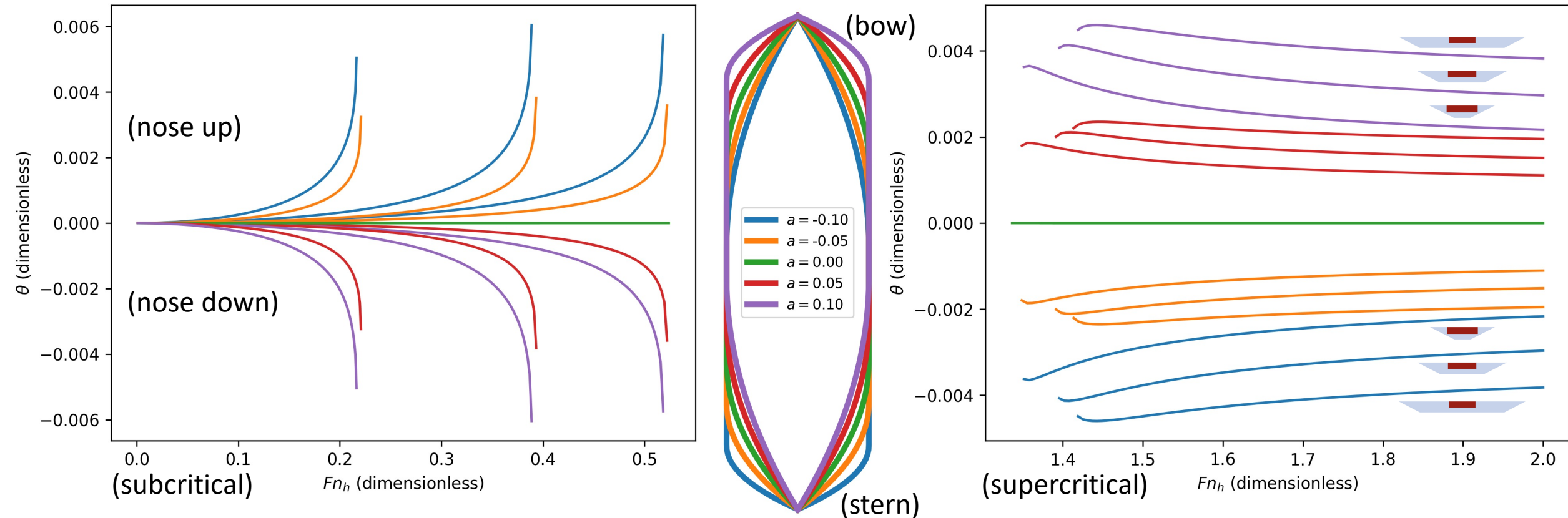
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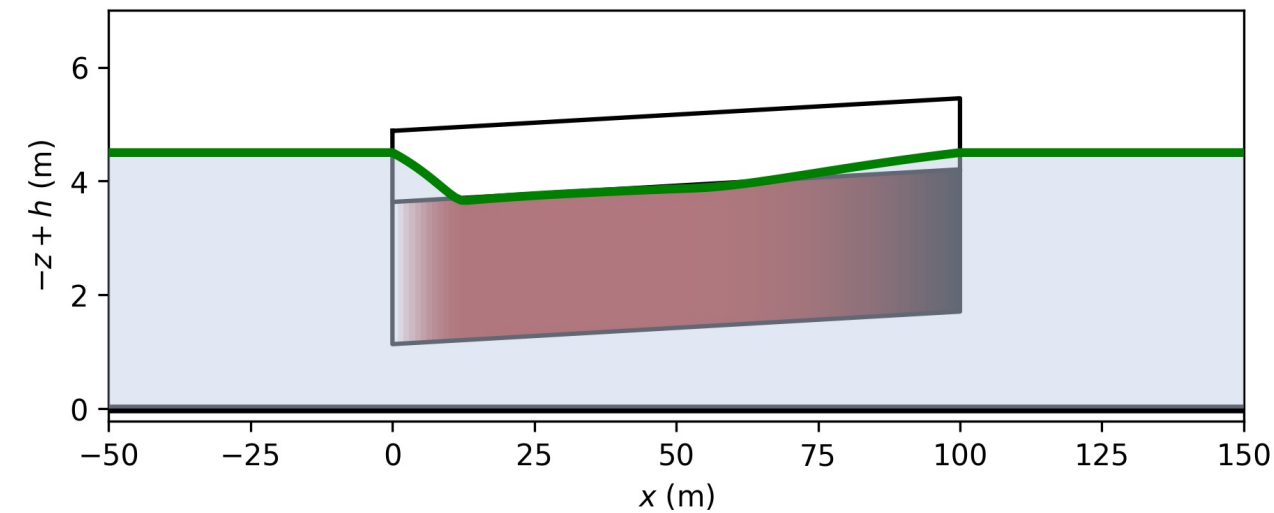
Numerics : the effect of a

$$C_b = 0.8, a = -0.1$$

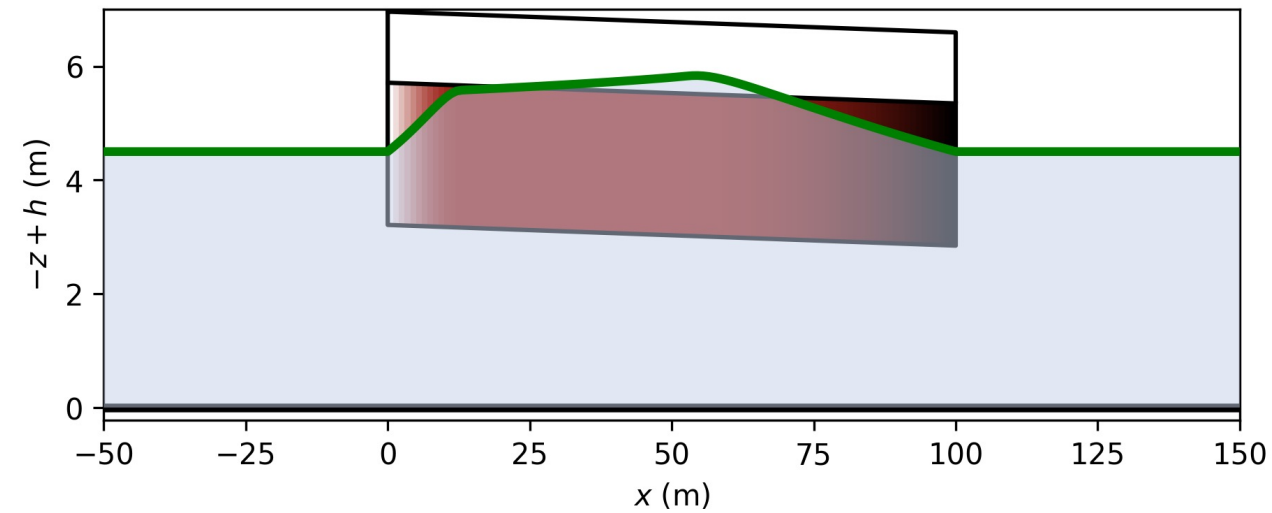
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$$Fn_h = Fn_h^-$$



$$Fn_h = Fn_h^+$$



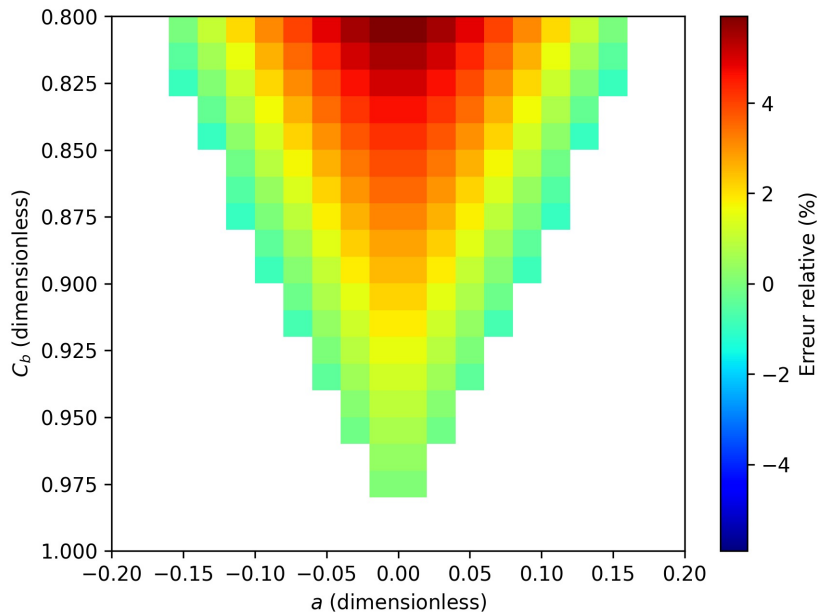
Comparison w/ t-KSC

Relative error between our theory and t-KSC on $F n_h^-$
 $a \in [-0.15, 0.15]$, $C_b \in [0.8, 1]$

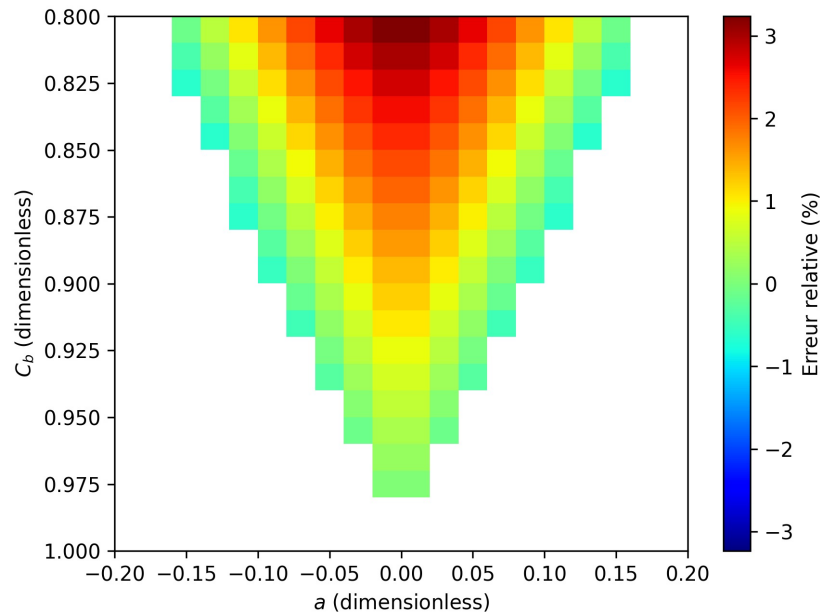
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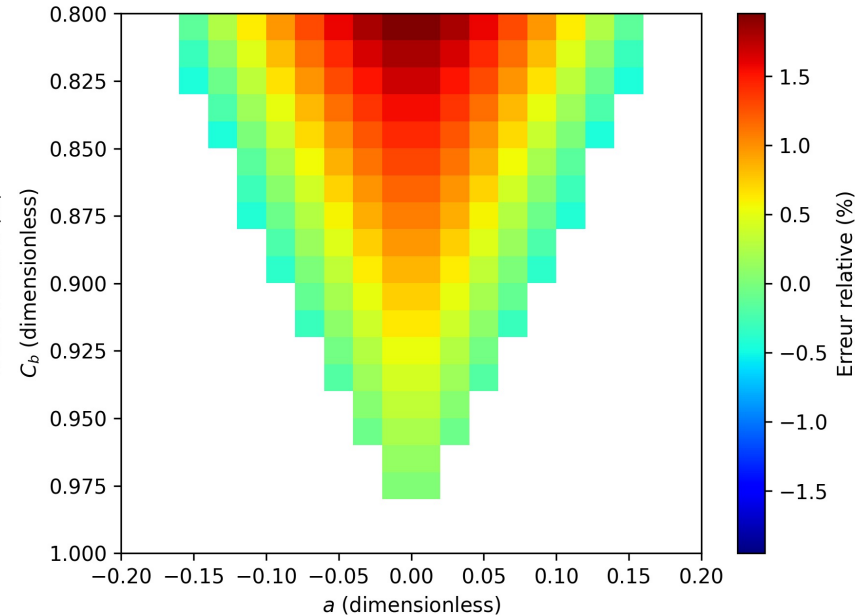
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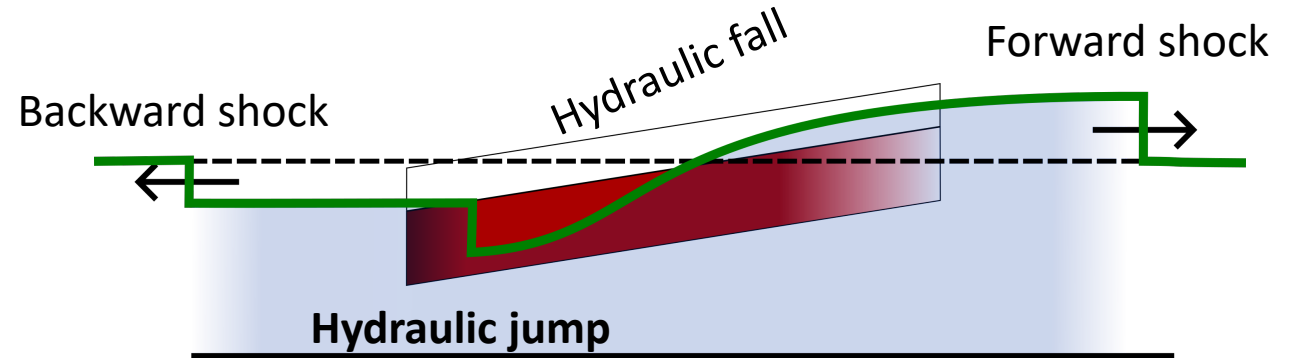


Conclusions

- Taking into account **end profiles** in stationary hydraulic models
- The predictions ($u, z, \bar{z}, \theta$) depend not only on Sr , An , but also on C_b and a
- As $C_b \rightarrow 1$, we retrieve the predictions of the t-KSC theory
- This effect is marginal when Sr is moderate (e.g. class Vb)...
- ...but larger when Sr is closer to 1 (*i.e.* when the model is accurate)
- The trim depends on **the asymmetry of the boat** :
 - Nose up when the boat is stern prone (subcritical regime)
 - Nose down when the boat is bow prone (subcritical regime)

Perspectives/Challenges

- Transcritical flows
 - **Hydraulic jump** position: head loss
 - Complex interaction with the boat
 - Trim even for $a = 0$
- Transverse flows
 - Getting rid of the $Sr \sim 1/p \sim 0$ hypothesis
 - 2D stationary Saint-Venant equation (SWE): **congestion problem !**
- Introducing dispersion
 - Wave wake
 - **Long-wave asymptotics** is only possible in the limit: $\mu \rightarrow 0, m \rightarrow 0, Fn_h \rightarrow 1$



Thank you for your attention !

